ADDITIONAL MATHEMATICS

Paper 0606/01

Paper 1

General comments

The examination produced a wide range of marks. In the past, the topic of vectors has posed problems for candidates; however, in this paper it was pleasing to see that more candidates were able to cope with the topic. Most candidates had the relevant algebraic skills to deal with the various questions set. However, in this session there were a significant number of candidates unable to deal adequately with the solution of trigonometric equations.

Comments on specific questions

Question 1

This question produced mixed results. Many candidates found parts **(i)** and **(ii)** difficult. The great majority of candidates were able to obtain the correct solution for part **(iii)** which was marginally easier than the preceding parts.

Some candidates chose to define the sets using their own sets of numbers and hence attempt to draw the relevant Venn diagrams. In general, these candidates tended to be very successful.

Question 2

Very few candidates were unable to make an attempt to obtain a three term quadratic equation and solve it. Most candidates were able to obtain the correct critical values, but were unable to obtain the correct inequalities. Of those that did, many gave their answer in the form of one inequality rather than two separate ones.

Answer: $x < -1$, $x > 2$.

Question 3

This was generally well answered with most candidates working with just the left-hand side of the identity, correctly combining fractions over a common denominator. A minority of candidates failed to expand $(1+\cos A)^2$ correctly, but most attempted to use $\cos^2 A + \sin^2 A = 1$ correctly. To complete the proof correctly, candidates were expected to have both the dominator and the numerator in a factorised form to enable cancelling of factors to achieve the final required result. Unfortunately many candidates were unable to make this final step, very often having expanded out the denominator, hence making it difficult to recognise the next step to make.

Question 4

This question was reasonable well done with most candidates obtaining some marks. Most were able to gain credit for the substitution of $x = 1$ and $x = -\frac{1}{2}$, but some incorrectly equated their results to zero rather than the given values of 3 and 6. A few candidates misunderstood the question and chose to substitute in values of 3 and 6 for *x*. Many candidates were unable to simplify their equations correctly, with many making either arithmetic or algebraic errors, but most were able to solve the resulting simultaneous equations.

Answers: –12, 8.

Question 5

(i) Many candidates were unsure of what a unit vector is. Many realised that the magnitude of the vector was important and went on to find it correctly. However many were then unsure as to what to do with this magnitude.

(ii) Most candidates realised that they had to equate like vectors and solve the resulting simultaneous equations. Arithmetic slips involving negatives occasionally caused problems for some candidates.

Answers: **(i)** $\frac{1}{13}(5\mathbf{i} - 12\mathbf{j})$; **(ii)** 9, 2.

Question 6

(i) Most candidates were able to obtain a correct quadratic equation and produce a correct solution.

(ii) Very few candidates appreciated that $x^{\frac{1}{2}}$ could replace t and most resorted to long, usually incorrect, algebraic steps, sometimes involving logarithms which lead nowhere. A few candidates obtained the correct results from legitimate methods, but many gained 'correct' solutions fortuitously.

Answers: **(i)** $\frac{1}{2}$, 5; **(ii)** 0.25, 2.5.

Question 7

(i) Very few correct solutions were seen. Most candidates 'misread' the question and although their expression was correct, it was not what was required. The most common answer was $4(x - 1.5)^2 - 6$, with other similar variations being produced.

(ii) Most candidates chose to do this part using calculus and usually produced correct solutions. Most of those that used their answer to part **(i)** used it correctly.

(iii) Most candidates were unable to obtain a correct solution, failing to realise what was being requested of them.

Answers: **(i)** $(2x - 3)^2 - 6$; **(ii)** (–1.5, –6); **(iii)** f(x) › –6.

Question 8

This question was found to be one of the most difficult on the paper. Provided the candidates realised that integration had to be performed twice, each time involving a constant of integration that had to be found, then most scored highly. Problems arose if candidates were unable to integrate the exponential term correctly or chose to differentiate instead of integrate. Many candidates performed the first integration correctly but chose to continue incorrectly by finding the equation of a straight line.

Answer: $y = \mathrm{e}^{-2x} + 5x - 10$.

Question 9

(i) This part of the syllabus has clearly been well taught by most Centres. Most candidates were able to gain full marks for this part; however a few candidates made errors involving the negativity of the second term which then had a consequent effect in part **(ii)**.

(ii) There were many correct solutions fro this part of the question. Some candidates knew what to do but made errors either involving signs or arithmetic slips. Most realised that when it came to finding *b* and *c*, 2 terms from the expansion were needed. There were also some candidates who chose to write out the entire expansion and pick out the relevant terms, this method being more prone to errors.

Answers: **(i)** $32-240x+720x^2$; **(ii)** 2, 9, –720.

Question 10

(a) (i) Most candidates were able to apply the correct order to the application of the functions.

(ii) Again, having obtained a correct solution to part **(i)** the great majority of candidates were able to produce a correct solution.

(b) (i) This part was very often omitted and of those candidates who attempted it, very few correct solutions appeared, again showing a general misunderstanding or lack of understanding of the range of a function. Common incorrect answers included solutions given in terms of *x* or equated to 4 or 4.69 (obtained from 4 + ln2).

(ii) Whilst many candidates were able to find the inverse function correctly, there were also many who were unable to, having problems with dealing with the inverse of a logarithmic function.

(iii) Very few correct solutions were seen. Most graphs had curves crossing the coordinate axes and curves that did not intersect, but most candidates realised the symmetry of the curves about the line $y = x$.

Answers: **(a)(i)** $3-\frac{x}{x+2}$, **(ii)** –1.75; **(b)(i)** $h(x) > 4$, **(ii)** 148.

Question 11

This question was found to be difficult.

(i) Many candidates spent far too long getting the equation into a form that could be solved. Having done that, many omitted to deal with the solutions obtained from the negative root of the basic equation. Many candidates also failed to deal with the double angle, tending to ignore it completely.

(ii) Common errors were to write cosec *y* as 1 + cot *y,* or $\cot^2 y$ as $1 - \text{cosec}^2 y$. However, those candidates that were able to use the correct identity usually made a reasonable attempt to solve the resulting quadratic equation and hence obtain correct solutions. Many candidates were unable to solve sin *y* = 1 correctly, giving extra incorrect solutions, such as 180° or 270°.

(iii) Many more candidates were able to give a correct solution to this type of question than in the past. Many however are still unable to deal correctly with compound angles, by stating incorrectly that $\sec\left(z+\frac{\pi}{2}\right)=\sec z+\sec\frac{\pi}{2}$. Many candidates were able to deal with radians but failed to give their answers to the required level of accuracy (3 significant figures for radian answers). Too many confused radians and degrees, choosing to use a mixture of both together.

Answers: **(i)** $x = 30°, 60°, 120°, 150°$; **(ii)** $90°, 221.8°, 318.2°$; **(iii)** $\frac{\pi}{6}, \frac{5\pi}{6}$.

Question 12 EITHER

Most candidates attempted this alternative.

(i) Most candidates were able to make a correct attempt at finding the stationary points using the quotient or product rule. The quotient rule was most commonly used and many were able to find the correct coordinates; however, many candidates thought that the solution $x = 0$ was inadmissible and thus failed to gain full marks. Other candidates found it difficult to obtain a correct *y*-coordinate when $x = -2$.

(ii) Many candidates failed to progress with this part probably due to the unstructured nature of the question. Most attempted to find the gradient of the normal correctly but were unable to go further, not realising that they were able to find the coordinates of a point on the line using the original curve equation. Many failed to realise that they had already done the work in part **(i)** to enable them to find the coordinates of *N*. Few candidates were able to structure their work and produce a correct solution.

Answers: (**(i)** (0,0), (–2, – 4) **(ii)** 2.75)

Question 12 OR

(i) Most candidates were able to make a good attempt at finding the stationary point and its nature provided they could deal correctly with the exponential term. Those that were unable to deal with it correctly were usually able to obtain method marks. Some candidates were unable to solve the equation $e^{x-2} = 2$, again showing that some candidates are unable to deal with inverse functions when applied to logarithms and exponentials.

(ii) Again most candidates were able to apply a correct method to finding the enclosed area but, instead of keeping terms exactly, resorted to the use of their calculators when applying limits and often then failed to realise what they were meant to be doing.

Answers: (**(i)** (2.69, 2.61), **(ii)** 9)

ADDITIONAL MATHEMATICS

Paper 0606/02
Paper 2

General comments

There was a wide spread of marks on this paper, with some candidates producing excellent, well set out and concise work. There were others who found the paper very demanding and were unable to make any progress on any of the questions. In **Question 12** the majority of candidates chose to answer the **EITHER** alternative and this was the best answered question on the paper. **Questions 4**, **7** and **11** were the least well answered questions.

Comments on specific questions

Question 1

Most candidates correctly calculated the inverse matrix $\mathbf{A}^{-1}$. Many candidates failed to follow the instruction in the question, used elimination instead of the inverse matrix and so were unable to gain the marks for solving the equations.

Answers: $\frac{1}{10}\begin{pmatrix} 4 & -6 \\ -7 & 13 \end{pmatrix}$; $x = 2$, $y = 2.5$.

Question 2

The majority of the weak candidates substituted $x = 7$, or $x = 4$ into the given formula and gained no marks. Many candidates, realising that differentiation was required, calculated $\frac{dy}{dx}$ correctly, but failed to connect this with requiring the rate of increase of *y*. Only the more able candidates realised that $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ with $x = 7$ and $\frac{dx}{dt} = 4$ was required. Common errors were to use $\frac{dx}{dt} = 3$ or to divide by 4 instead of multiplying by 4.

Answer: 600 units per second.

Question 3

The majority of candidates equated the two expressions and used $b^2 - 4ac$. The most common error was to make a mistake in sign and to arrive at $m^2 - 10m - 39$. Some were confused and solved $x^2 - 5x + 18 = 0$ and used these values of *x* as their values of *m.*

Answer: $-13 < m < 3$.

Question 4

In part **(i)** many correct answers were seen, but a common mistake was to simplify $x \times \frac{1}{x} + \ln x$ to 0 + ln *x* giving a final answer of ln *x*. Failure in part **(i)** led to difficulty in part **(ii)**. Only a few candidates seemed to understand the implication of the word 'hence' and the majority failed to connect part **(ii)** with part **(i)**; many tried to integrate from scratch giving incorrect answers such as $\frac{\ln x^2}{2}$. Many good candidates realised that $\int (1 + \ln x)\,dx = x \ln x$ and went on to write $\int \ln x\,dx = x \ln x - 1$ without appreciating the need to integrate the 1.

Answers: **(i)** 1 + ln *x*; **(ii)** – *x* + *x* ln *x* + *c*.

Question 5

Part **(i)** was generally done better than part **(ii)** where even some very strong candidates did not reject the answer of –10. In part **(i)** most candidates knew to express 4^x and 8^{x-3} as powers of 2 but writing $8^{x-3} = 2^{3x-3}$ was very common. Of those who correctly reached $\frac{2^{2x}}{2^{5-x}} = \frac{2^{4x}}{2^{3x-9}}$ many proceeded to write $\frac{2x}{5-x} = \frac{4x}{3x-9}$. Many candidates made no attempt to answer part **(ii)**, but some who did made little progress, writing $\lg(2y+10) + \lg y = \lg 2y + \lg 10 + \lg y$, and others did not use 2 = lg 100.

Answers: **(i)** 7; **(ii)** 5.

Question 6

In part **(a)**, 28 was the most common wrong answer seen, as candidates thought they had to add the 10, 3 and 15 rather than multiply them. Part **(b)** was found harder by candidates; 360 was a common incorrect answer and many candidates used 3 instead of 4 as they did not realise they could use the 3 as the first digit.

Answers: **(a)** 450; **(ii)** 240.

Question 7

Weaker candidates mixed distance and speed and produced calculations involving 1.4 and 48. Some candidates who scored very few marks on the paper found the speed of 4.8 m/s; some then simply added or subtracted 4.8 and 1.4. Correct diagrams were quite rare but were sometimes implied. Some candidates who produced an incorrect diagram still calculated the correct speed of 5 m/s.

Answers: **(i)** 5 ms^{-1}; **(ii)** 73.7°.

Question 8

The amplitude was commonly given as 2 or 3 instead of 5 and the values 2, 3 or 5 often appeared in the answer to the period. Many candidates embarked on differentiation to find the maximum and minimum values of f and were usually not successful. The values given in part **(iii)** often bore no relation to those on the sketch, where the correct values of –2 and 8 were frequently found.

Whilst the graph usually started at (0, 3) it often finished at (2π, 0) and not all candidates drew the graph for all the domain, often stopping at π. However the majority of candidates could draw a reasonable sine curve, even if it was located incorrectly.

Answers: **(i)** 5; **(ii)** π, **(iii)** 8, –2.

Question 9

The algebra in this question was often very impressive but many candidates omitted to find the mid-point of *AB* and used the coordinates of either *A* or *B* in the equation $y = -\frac{1}{2}x + c$.

Answer: $x + 2y + 3 = 0$.

Question 10

The candidates were asked to show that the minimum occurred at the point (4, 0) in order to ensure that part **(ii)** was accessible to all candidates. Unfortunately in part **(i)** some candidates merely confirmed that the point (4, 0) was on the curve. Others did differentiate and found the values of x as 0 and $\frac{4}{3}$ but did not find the corresponding minimum and maximum values of y. Others substituted $x = 4$ into $\frac{dy}{dx}$ and proved that $x = 4$ was where the minimum occurred, but did not confirm the minimum value was 0 and did not find the maximum at all. Another common mistake was merely to solve $y = 0$ and obtain the values of $x = 0$ and $x = 4$.

Part **(ii)** was usually done well, however, some candidates who had integrated the function correctly used $x = \frac{4}{3}$ as the lower limit. Weaker candidates were not sure whether to differentiate or to integrate, and there were often errors in the coefficients.

Answers: **(i)** $\left(\frac{4}{3},\ 9\frac{13}{27}\right)$; **(ii)** $21\frac{1}{3}$.

Question 11

Weak candidates made no progress at all, often plotting y against x, or xy against $\frac{1}{x}$ but with $\frac{1}{8}, \frac{1}{6}, \frac{1}{4}, \frac{1}{2}$ plotted at equal intervals. A few just plotted the four given values with equal intervals for y, i.e. 2.25, 0.81, 0.47, 0.33, sometimes in that order from bottom to top. Even when the correct variables were plotted, the accuracy of plotting varied and some candidates drew graphs with totally unsuitable scales.

Only stronger candidates made a reasonable attempt at part **(ii)**. Among these, the gradient was quite often found and the intercept was calculated or, very occasionally, read off. There were many mistakes in this part, usually involving reading from the graph or arithmetical mistakes. Quite a few candidates gave the reciprocal of the gradient. Many candidates wrote $y = 5x + 2$, or stopped at $xy = \frac{5}{x} + 2$ without making y the subject.

For part **(iii)**, many candidates found $\frac{1}{x} = 0.4$ but quite a few stopped there. Amongst those who tried an algebraic approach, often using $xy = \frac{5}{x} + 2$, there were many slips in algebraic rearrangement.

Answers: **(ii)** $y = \frac{5}{x^2} + \frac{2}{x}$; **(iii)** 2.5, 1.6.

Question 12 EITHER

This was by far the more popular alternative chosen for this final question and in general candidates scored highly on it. Even candidates who felt they had to change angle *AOB* into degrees first usually scored the marks in part **(i)**, although some of these candidates rounded inappropriately and lost accuracy marks in this and subsequent parts. In part **(ii)** most candidates knew they needed to use $s = r\theta$ to find the length of arc *AB*. A common error was to subtract the perimeter of triangle *OCD* from the perimeter of sector *OAB*. Part **(iii)** was answered well, even by those who had made errors in part **(ii)**.

Answers: **(i)** 1.13 cm; **(ii)** 13.1 cm; **(iii)** 9.87 cm^2.

Question 12 OR

This alternative was not chosen very often but there were some good answers from strong candidates and only the very weak candidates could not start it, having no idea that it was a calculus question. In part **(i)** it was common to see the constant of integration omitted and then $2t^2 - 12t$ equated to either 16 or 0. In part **(ii)** most candidates had only the first two terms to work with and the 'distance travelled in the fifth second' was not always understood, with only the value $t = 5$ used in many cases. The correct use of limits was rare.

Answers: **(i)** 2, 4; **(ii)** $2\frac{2}{3}$ m.